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MATHEMATICAL MODELS FOR DETERMINING THE PROTECTED SPACES OF THE VERTICAL LIGHTNING ROD

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ABSTRACT

This paper is concerned with two mathematical models for determining the protected spaces of the vertical lightning-rod. In the first model there was applied the circular approximation. Through the introduction of the modified striking distance in the second improved approximation there was obtained a new model for the protected space of the lightning-rod. The models are of general type, foreseen for the three-dimensional space and they are simply applied on solving the practical problems.

INTRODUCTION

The new theories about a greater number of protected spaces of one vertical lightning-rod depending on the maximum value of the stroke current impulse afford the possibility of a more precise evaluation of the lightning-rod's protective effect. The models to be set out in this paper start from the approach to the determination of protected spaces around the lightning-rod, depending on the amplitude of the stroke currents [1].

Namely, the degree of menace is evaluated on the basis of the determination of maximum stroke currents to which is exposed a structure of a smaller height or a device in the proximity of the vertical lightning-rod or of a structure playing a role of the lightning-rod. The procedure is based on the determination of protected space, which is a function of the well-known striking distance, the height of the lightning-rod, the height of the protected structure and their mutual distance. The length of the striking distance, for the purpose of the maximum stroke current, is determined by electrogeometrical method and numerically. In further elaboration there will be discussed two approximative approaches - the circular approximation of protected spaces and the elliptical approximation of protected spaces, constituting a more precise determination of the protected space, where was taken into consideration the impact of change in the electric field due to the presence of the lightning-rod. The above approximations are spaciously enlarged because of the more precise evaluation of the degree of menace, taking into account all the structure's dimensions. There are also given the corresponding mathematical models for both approximations.

CIRCULAR APPROXIMATION MODEL

When the leader of the atmospheric electrical discharge is approaching a very long vertical lightning-rod or a high structure, the site of striking will be a point on the structure nearest to the top of the coming leader, i.e. when the distance equals the striking distance. In Fig. 1 is shown an example when the critical electric field is equal in two points and the thunderbolt is possible on any of them.

In this model we start from the assumption that the generating line of "cone"[1] is the arc of circle of a radius approximately equalling the striking distance. Such an approximation is satisfactory in case when we have in view the fact that the striking distance depends exclusively on the stroke current, and when we neglect the other influential factors.

In this case the protected space is within the "cone", whose generating line is the arc of circle. The height of the cone is the height of the lightning-rod or of the high structure H, and the radius of the basis equals approximately the striking distance D.

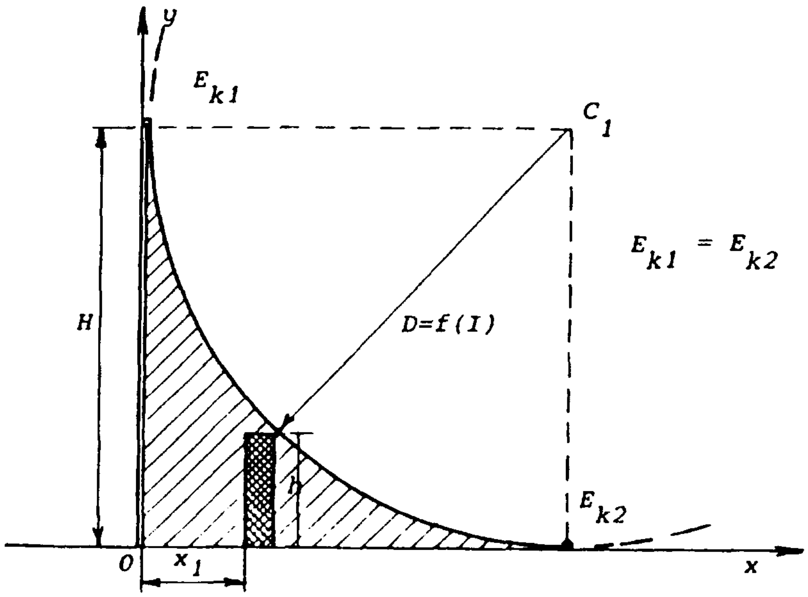

Fig. 1 The geometrical determination of the striking distance

The geometrical determination of the striking distance for this case is reduced to the construction of the circle attaining the protected structure and passing through the top of the lightning-rod. For numerical calculation of the striking distance we start from the following equation:

$$(x-D)^2 + (y-D)^2 = D^2 \tag{1}$$

provided that $x \le D$ and $y \le H$.

The practical application of this model is presented in two examples when the height of the vertcal lightning-rod is smaller than the striking distance (H<D) and when the height of the vertical lightning-rod is greater than the striking distance (H>D).

The first case is illustrated in Fig. 2 and it practically refers to the small (low) lightning-rods of the height H in whose close proximity is, at the distance r, the structure of the height h (h>H).

Through the application of (1) we obtain:

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[1] *The "cone" stands for a surface of revolution obtained by the revolution of the arc around the lightning-rod's axis to the curve of which corresponds the radius approximating the striking distance.*

$$D = \frac{H+h}{2(H-h)^2}\left[(H-h)^2+r^2\right] + \frac{Hr}{(H-h)^2}\sqrt{\frac{h}{H}\left[r^2+(H-h)^2\right]} \qquad (2)$$

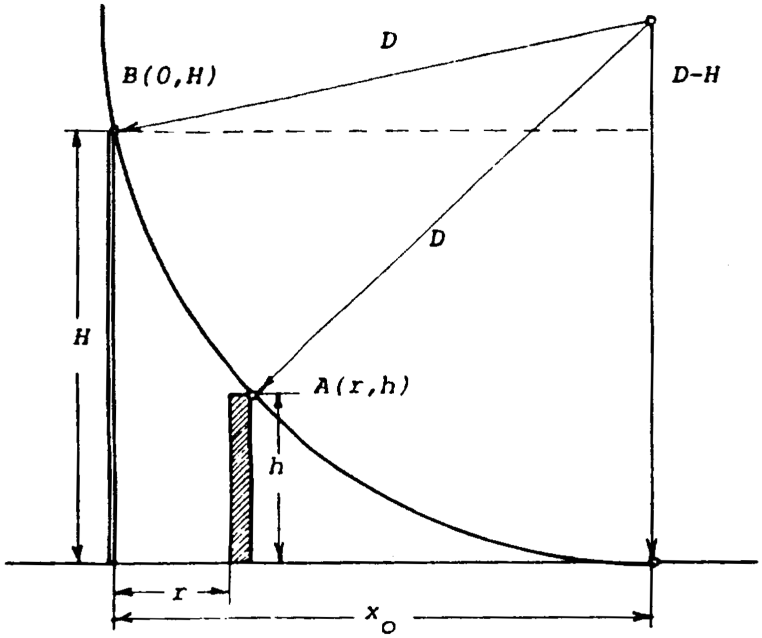

Fig.2 The case H<D

In Fig. 3 is represented another case (H>D). This is the case of the very high lightning-rods of the height H, or of the high structure whose slenderness permits it to be considered a lightning-rod. Here we can notice a part of the inefficient height of the lightning-rod for the currents of lower intensity.

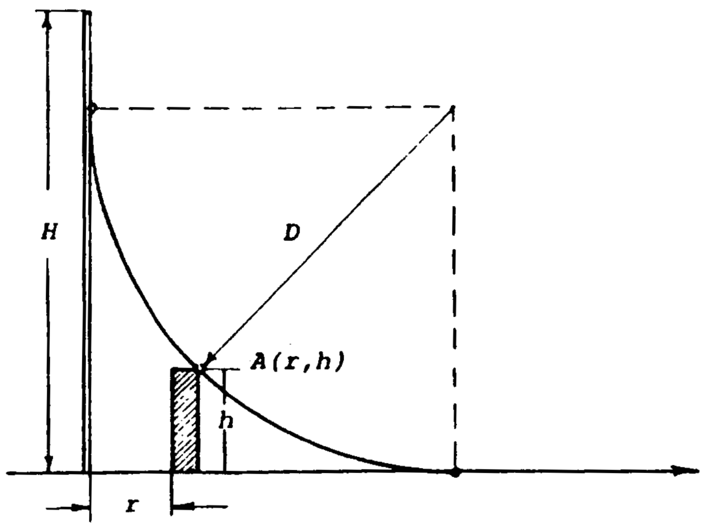

Fig.3 The case H>D

According to (1), the striking distance D is given by the expression:

$$D = (r+h) \pm \sqrt{2rh} \qquad (3)$$

In addition to the above examples, this model may be also applied in the case when the other dimensions of the structure are taken into consideration [2].

ELLIPTICAL APPROXIMATION MODEL

The value of the striking distance, in addition to the maximum value of the stroke current (exerting the greatest influence), is influenced by the other parameters, too. In his research work Whitehead [3] found out that the height of the structure, i.e. the height of the lightning-rod because of the increased value of the electric field at the top, exerted the influence on the value of the striking distance. He introduced the modified striking distance (D^*) and gave the following functional dependence in relation to the reference striking distance towards the ground:

$$D^* = D\,[1+E_1 I_f{}^2 \exp(-D/H)] \qquad (4)$$

where:

D^* - modified striking distance to a structure of height H,

E_1 - field intensification factor, which takes into account the structure the slenderness ratio,

I_f - factor which relates the prospective stroke current to the median of a reference current amplitude distribution,

D - reference striking distance to the ground, given by

$$D = 10\ I^{0.65} \qquad (5)$$

Having in view the modified striking distance which is, according to the expression (4), greater than the reference one, in this approximative approach we started from the fact that the protected space of the vertical lightning-rod (a high structure) of the height H is within the "cone with the elliptical generating line" - Fig. 4.

The basis of such an obtained "cone" is a circle, the radius of which equals approximately the length of the modified striking distance.

The case in Fig. 4 may be considered to be characteristic (a limit one) with the current I_1 and D=H. Observed in the xOy plane, this is an ellipse with semiaxes D^* and D, and the equation is:

$$\frac{(x-D^*)^2}{D^{*2}} + \frac{(y-D)^2}{D^2} = 1 \qquad (6)$$

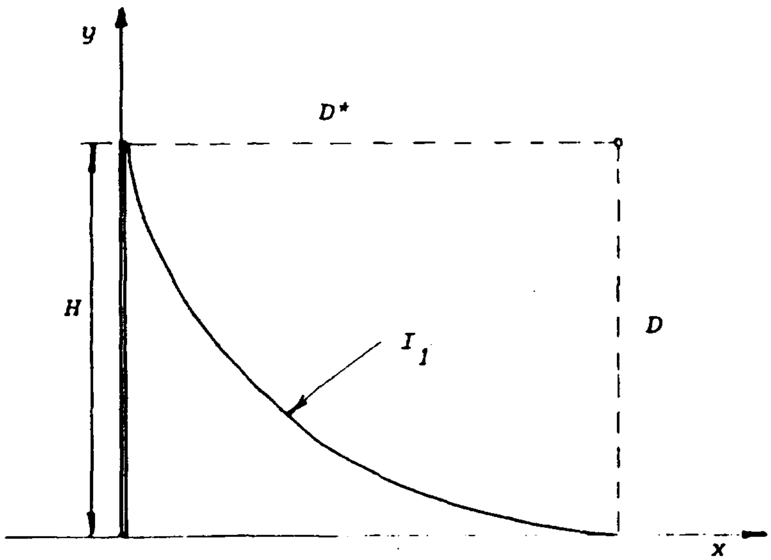

Fig.4 The protective zone with modified striking distance

As in the previous model, here too, we can discuss two cases. The first case refers to the protected spaces of those currents whose values are smaller than the values of the limit current I_1, i.e. $I_1'<I_1$. The graphic illustration of this case is displayed in Fig. 5, with the generating lines of "cone" within which are the protective zones, the arcs of ellipses:

$$\frac{(x-D_i^*)^2}{D_i^{*2}} + \frac{(y-D_i)^2}{D_i^2} = 1 \qquad i=1,2,3,\ldots \qquad (7)$$

where:
D_i^* and D_i are the modified and reference striking distances for the current less powerful than $I_1 (I_i<I_1)$.

The second case refers to the striking distances D^* and D greater than the height of the lightning-rod ($D^*>D>H$), i.e. for the currents more powerful than $I_1 (I_i>I_1)$. In Fig.6 is displayed the illustration of this case in the xOy plane. The equation of any curve from Fig.6, i.e. for any current $I_i (I_i>I_1)$ and the corresponding distances D_i^* and D_i reads as follows:

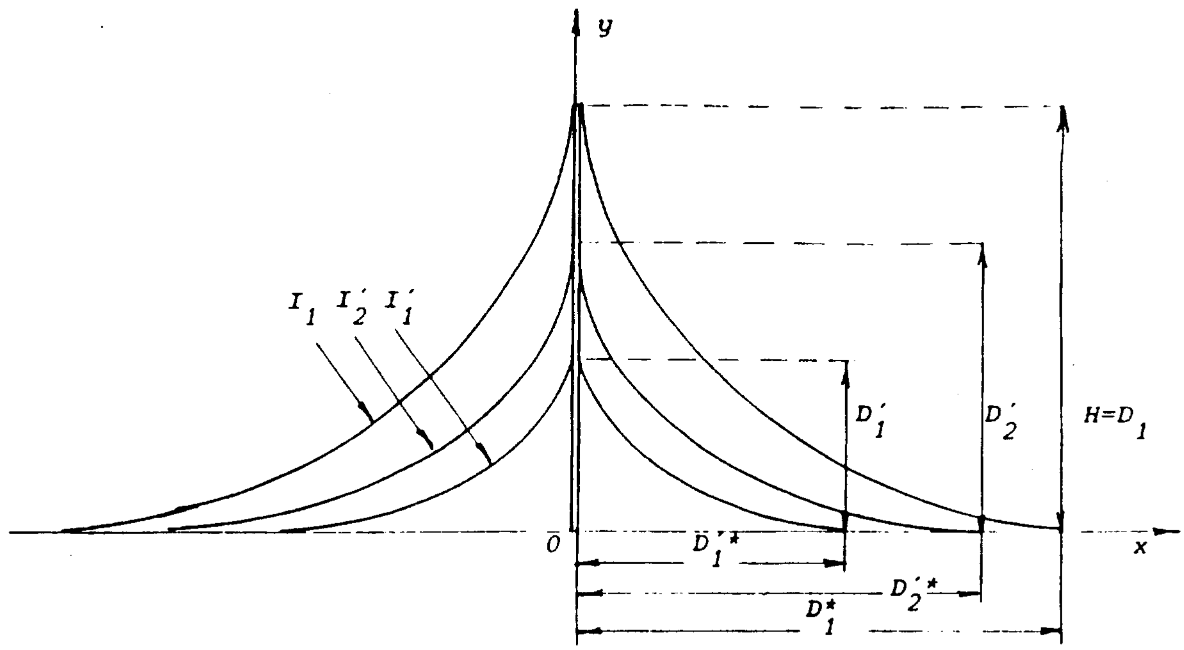

Fig.5 The protective zones for the case $H>D>D^*(I_1>I'_2>I'_1)$

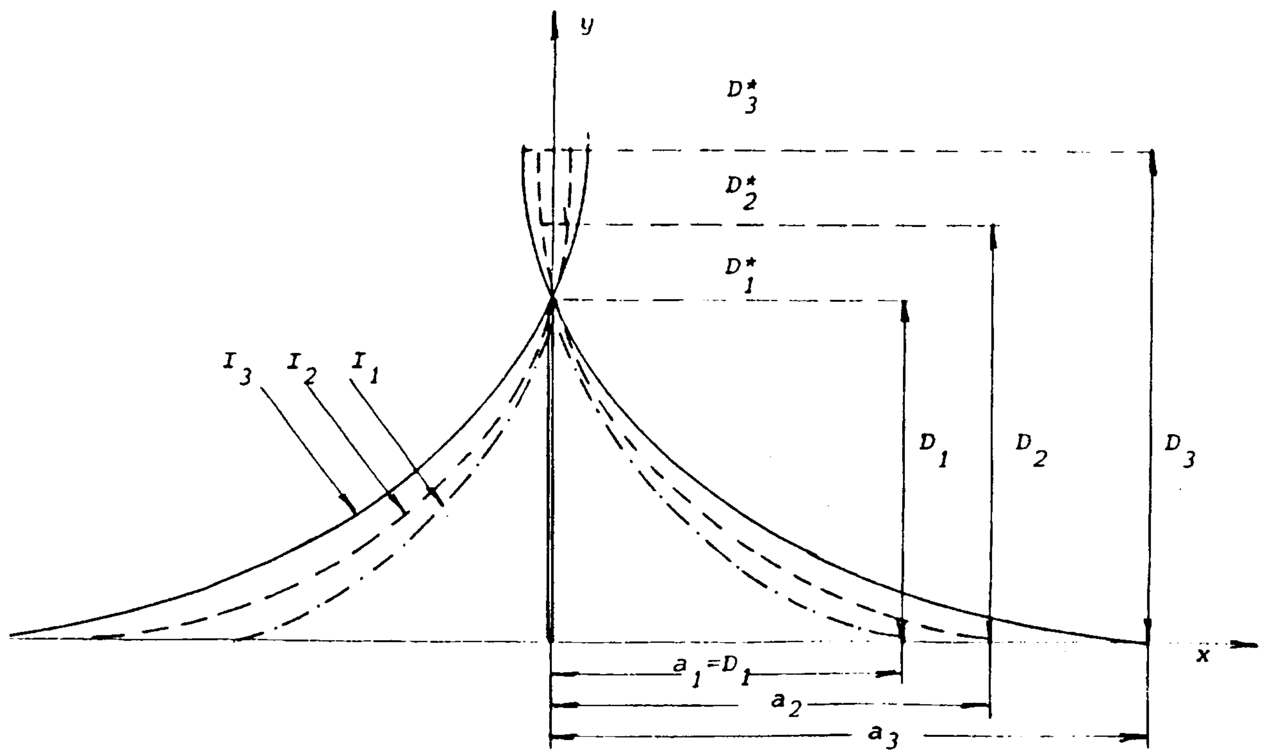

Fig.6 The case $D^*>D>H$ $(I_3>I_2>I_1)$

$$\frac{(x-a_i)^2}{D^{*2}} + \frac{(y-D_i)^2}{D_i^2} = 1 \qquad (8)$$

where:

$$a_i = \frac{D_i^*}{D_i} \sqrt{H(2D_i-H)} \qquad (9)$$

provided that $D_i^* > D_i > H$.

CONCLUSION

The mathematical models set out in this paper afford the possibility of electrogeometrical and analytical determination of the protective spaces of the vertical lightning-rod or of the high structure which may play the role of a lightning-rod.

By set out methods we can evaluate the degree of menace of a structure in the close proximity of the high structure or of the vertical lightning-rod. The reason for this is the fact that the analysis may be carried out within the space with the influence of all the dimensions of the protected structure and that through the modified striking distance are measured the influences of several parameters of lightning on the protective zone.

The purpose of this paper is to point to the new electrogeometrical method and to the corresponding mathematical models for as precise as possible determination of the striking distance. The application of the set out models within the space and to the practical examples is presented in [2].

REFERENCES

[1] Vorgučić A.: Condition for Evaluation of the Protective Zone of the Lightning Rod, FAA_RD_78_83, Atlanta (USA), pp. 115-129, 1978.

[2] Mladenović I.: Stepen ugroženosti od udara groma uredjaja i objekta u blizini vertikalnog gromobrana, Doktoral dissertation, Faculty of Electronic Engineering, Niš, (YU), pp. 60-81, 1988.

[3] Whitehead, E.R.: Analytical speculation on Improved Geometric Models of the Lightning Flash and Transmission Line Environment and Discussion, Private Communications, August 1976 and June 1977.